1 Key Results Without Limited Liability

The results of the paper do not depend on the limited liability assumption of the insurer. To see this, let the payoff of the IFI be given by the following.

$$\begin{aligned}\Pi_{IFI} &= (1-b)\left[\int_{-P\gamma(\beta+(1-\beta)R_I)}^{\overline{R}_f} \theta f(\theta)d\theta + \int_{\underline{R}_f}^{-P\gamma(\beta+(1-\beta)R_I)} (\theta - G)f(\theta)d\theta\right] \\ &\quad + (b)\left[\int_{C(\gamma-\beta P\gamma)}^{\overline{R}_f} (\theta - C(\gamma-\beta P\gamma) - \beta P\gamma)\, f(\theta)d\theta + \int_{\underline{R}_f}^{C(\gamma-\beta P\gamma)} (\theta - G)\, f(\theta)d\theta\right] \\ &\quad + P\gamma(\beta + (1-\beta)R_I) \qquad (1)\end{aligned}$$

There are 2 key differences to the setup with limited liability. First, the payoff can now be negative, and second, there is a bankruptcy cost G. Note that this bankruptcy cost can be zero as will be discussed below.

The first term is the expected payoff when a claim is not made, which happens with probability $1-b$ given the IFI's beliefs. The $-P\gamma(\beta+(1-\beta)R_I)$ term in the integrand represents the benefit that engaging in these contracts can have: it reduces the probability of portfolio default when a claim is not made. We assume that $\underline{R}_f$ is sufficiently negative so that $P\gamma\left(\beta+(1-\beta)R_I\right) < |\underline{R}_f|$. Since P and β are both bounded from above[1], it follows that this inequality is satisfied for a finite $\underline{R}_f$. This assumption ensures that the IFI cannot completely eliminate its probability of default in this state. Recall that before the IFI engaged in the insurance contract, they would be forced into insolvency when their portfolio draw was less than zero. However, if a claim is not made, they can receive a portfolio draw that is less than zero and still remain solvent (so long as their draw is greater than $-P\gamma(\beta+(1-\beta)R_I)$).

The second term is the expected payoff when a claim is made, which happens with probability b given by the IFI's beliefs. The term $C(\gamma-\beta P\gamma)$ represents the cost to the IFI of accessing the needed capital to pay a claim. Notice that the loans placed in the illiquid asset are not available if a claim is made. Furthermore, the probability of default for the IFI increases in this case. To see this, notice that before engaging in the insurance contract, the IFI defaults if its portfolio draw is $\tilde{\theta} \in [\underline{R}_f, 0]$. After they sell the insurance contract, they default if the draw is $\tilde{\theta} \in [\underline{R}_f, C(\gamma-\beta P\gamma) > 0]$. To ensure that the IFI prefers to pay the insurance contract when they are solvent, we assume that $G \geq C(\gamma-\beta P\gamma) + \beta P\gamma$. Intuitively, if this condition does not to hold, the IFI would rather declare bankruptcy than fulfil the insurance contract, no matter what their portfolio draw is. This condition arises because the IFI receives the payoff from the invested premium if they fail. Alternatively, we could force the liquidation of the IFI in the bankruptcy case and set $G=0$. The final term in (1) $(P\gamma(\beta+(1-\beta)R_I))$ is the payoff of the insurance premium given how it was invested.

We now re-state the key results of paper. The proofs (if applicable) can be found in the Appendix.

[1] This is true for β by construction and will be proven for P in Lemma 2.

Lemma 1 *The optimal investment in the liquid asset (β^*) is weakly increasing in the belief of the probability of a claim (b). Consequently, $\beta_R^* \geq \beta_S^*$.*

Lemma 2 *The market clearing price exists in the open set $(0, 1)$ and is unique.*

Lemma 3 *The market clearing price P^* is increasing in the belief of the probability of a claim (b). Consequently, $P_R^* > P_S^*$.*

Lemma 4 *If b decreases, but the actual probability of a claim does not, counterparty risk rises whenever $\beta \in (0, 1]$.*

Proposition 1 *In the absence of counterparty risk, no separating equilibrium can exist. When there is counterparty risk, the moral hazard problem allows a unique separating equilibrium to exist in which each bank type truthfully announces its loan risk. Sufficient conditions for uniqueness include 1) the safe loan is relatively safe and 2) Z is large enough. Formally,*

1. $$\left(p_s : p_s \geq 1 - \frac{P_R^* - P_S^*}{(1+Z)\int_{C(\gamma-\beta_R^* P_R^* \gamma)}^{C(\gamma-\beta_S^* P_S^* \gamma)} dF(\theta)} \;\&\; p_s > 1 - \frac{P_{1/2}^* - P_S^*}{(1+Z)\int_{C(\gamma-\beta_{1/2}^* P_{1/2}^* \gamma)}^{C(\gamma-\beta_S^* P_S^* \gamma)} dF(\theta)} \right)$$

2. $$\left(Z : Z \geq \frac{P_R - P_S}{(1-p_R)\int_{C(\gamma-\beta_R^* P_R^* \gamma)}^{C(\gamma-\beta_S^* P_S^* \gamma)} dF(\theta)} - 1 \;\&\; Z > \frac{P_R^* - P_{1/2}^*}{(1-p_R)\int_{C(\gamma-\beta_R^* P_R^* \gamma)}^{C(\gamma-\beta_{1/2}^* P_{1/2}^* \gamma)} dF(\theta)} - 1 \right).$$

Proof. As in the case with limited liability.

Proposition 2 *Any competitive equilibrium characterized by an investment decision $\beta^* \in (0, 1]$ is inefficient.*

Lemma 5 *For a given aggregate shock, there is less counterparty risk when the IFI's beliefs put more weight on the aggregate shock being risky ($q_A = r$) as opposed to it being safe ($q_A = s$).*

Proposition 3 *There exists a parameter range in which a unique separating equilibrium in the aggregate shock can be supported.*

Proof. As in the case with limited liability.

2 Appendix

Proof of Lemma 1. Using the assumption that $f(\theta)$ is distributed uniform over the interval $[\underline{R}_f, \overline{R}_f]$, we solve for the optimal choice of β for the IFI, given b and P.

$$\max_{\beta \in [0,1]} \Pi_{IFI}$$

Using Leibniz rule to differentiate the choice variable in the integrands, we obtain the following first order equation:

$$
\begin{aligned}
0 &= \frac{bP\gamma}{\overline{R}_f - \underline{R}_f}\left[C'(\gamma-\beta P\gamma)\left(G - C(\gamma-\beta P\gamma) - \beta P\gamma\right) + \left(\overline{R}_f - C(\gamma-\beta P\gamma)\right)\left(C'(\gamma-\beta P\gamma)-1\right)\right] \\
&\quad +(1-b)\frac{G}{\overline{R}_f - \underline{R}_f}\left[-R_I\gamma P + \gamma P\right] + P\gamma(1-R_I) \qquad (2)
\end{aligned}
$$

Where $G - C(\gamma-\beta P\gamma) - \beta P\gamma \geq 0$ by assumption, and $C'(\gamma-\beta P\gamma) - 1 \geq 0$ since $C(x) \geq x \ \forall\ x \geq 0$. To ensure a maximum, we take the second order condition and show the inequality that must hold.

$$
\begin{aligned}
C''(\gamma-\beta P\gamma)\left(G - C(\gamma-\beta P\gamma) - \beta P\gamma\right) + \left(\overline{R}_f - C(\gamma-\beta P\gamma)\right) C''(\gamma-\beta P\gamma) \\
\geq 2C'(\gamma-\beta P\gamma)\left(C'(\gamma-\beta P\gamma)-1\right) \qquad (3)
\end{aligned}
$$

Note that this holds with equality when $C(x) = x\ \forall x \geq 0$ so that $C'(x) = 1\ \forall x \geq 0$ and $C''(x) = 0\ \forall x \geq 0$. Plugging in the boundary conditions for β into the FOC, we now derive the optimal proportion of capital put in the liquid asset as an implicit function.

$$
\begin{cases}
\beta^* = 0 & \text{if } b \leq b^* \\
-(1-b)(R_I-1)G + b[C'(\gamma-\beta^*P\gamma)\left(G - C(\gamma-\beta^*P\gamma) - \beta^*P\gamma\right) & \\
+\left(\overline{R}_f - C(\gamma-\beta^*P\gamma)\right)\left(C'(\gamma-\beta^*P\gamma)-1\right)] = (R_I-1)(\overline{R}_f - \underline{R}_f) & \text{if } b \in (b^*, b^{**}) \\
\beta^* = 1 & \text{if } b \geq b^{**}
\end{cases}
$$

where $b^* = \frac{(R_I-1)(G+\overline{R}_f-\underline{R}_f)}{G(R_I-1)+C'(\gamma)(G-C(\gamma))-(\overline{R}_f-C(\gamma))(C'(\gamma)-1)}$,
and $b^{**} = \frac{(R_I-1)(G+\overline{R}_f-\underline{R}_f)}{G(R_I-1)+C'(\gamma-P\gamma)(G-C(\gamma-P\gamma)-P\gamma)-(\overline{R}_f-C(\gamma-P\gamma)))(C'(\gamma-P\gamma))-1)}$.

We now show that the optimal proportion of capital put in the liquid asset is increasing in b by finding $\frac{\partial\beta}{\partial b}$ from the FOC.

$$
\begin{aligned}
0 &= A + b[(-C'(\gamma-\beta P\gamma)(-\frac{\partial\beta}{\partial b}P\gamma)(C'(\gamma-\beta P\gamma)-1) \\
&\quad +(\overline{R}_f - C(\gamma-\beta P\gamma))(C''(\gamma-\beta P\gamma)(-\frac{\partial\beta}{\partial b}P\gamma) \\
&\quad +C''(\gamma-\beta P\gamma)(-\frac{\partial\beta}{\partial b}P\gamma)(G - C(\gamma-\beta P\gamma) - \beta P\gamma) + C'(\gamma-\beta P\gamma)(-C'(\gamma-\beta P\gamma)(-\frac{\partial\beta}{\partial b}P\gamma) \\
&\quad -(-\frac{\partial\beta}{\partial b}P\gamma))] + G(R_I-1)P\gamma \qquad (4)
\end{aligned}
$$

Where we define:

$$
A = C'(\gamma-\beta P\gamma)P\gamma\left(G - C(\gamma-\beta P\gamma) - \beta P\gamma\right) + \left(\overline{R}_f - C(\gamma-\beta P\gamma)\right)P\gamma\left(C'(\gamma-\beta P\gamma)-1\right) \geq 0. \quad (5)
$$

Assuming an interior solution and rearranging for $\frac{\partial\beta}{\partial b}$ yields to following.

$$\begin{aligned}\frac{\partial\beta}{\partial b} &= \frac{-C'(\gamma-\beta P\gamma)\left(G-C(\gamma-\beta P\gamma)-\beta P\gamma\right)-\left(\overline{R}_f-C(\gamma-\beta P\gamma)\right)\left(C'(\gamma-\beta P\gamma)-1\right)-G(R_I-1)}{-C''(\gamma-\beta P\gamma)\left(G-C(\gamma-\beta P\gamma)-\beta P\gamma\right)-\left(\overline{R}_f-C(\gamma-\beta P\gamma)\right)C''(\left(\overline{R}_f-C(\gamma-\beta P\gamma)\right))+2C'(\gamma-\beta P\gamma)\left(C'(\gamma-\beta P\gamma)-1\right)} \\ &> 0 \qquad (6)\end{aligned}$$

Where the numerator is trivially negative while the denominator is negative because of condition (3) imposed by the SOC to achieve a maximum.

■

Proof of Lemma 2.

Step 1: Existence

We prove that there exists a P^* that satisfies the following:

$$\begin{aligned}0 &= (1-b)\left[\int_{-P^*\gamma(\beta+(1-\beta)R_I)}^{0} Gf(\theta)d\theta\right]-b\left[\int_{C(\gamma-\beta P^*\gamma)}^{\overline{R}_f}\left(C(\gamma-\beta P^*\gamma)+\beta P^*\gamma\right)f(\theta)d\theta\right] \\ &\quad -b\left[\int_0^{C(\gamma-\beta P^*\gamma)} Gf(\theta)d\theta\right]+P^*\gamma(\beta+(1-\beta)R_I). \qquad (7)\end{aligned}$$

Consider $P^* \leq 0$. In this case, the IFI earns negative profits. To see this, notice all terms on the right hand side of (7) are weakly negative, with the second and third terms strict (since $C(\gamma-\beta P^*\gamma) > \beta P^*\gamma$ when $P^* \leq 0$). Therefore, it must be that $\Pi_{IFI}(\beta^*, P^* \leq 0) < 0$. This contradicts the fact that $\Pi_{IFI}(\beta^*, P^*) = 0$ in equilibrium.
Next, consider $P^* \geq 1$, and $\beta = 1$ (not necessarily the optimal value). In this case, the first term on the right hand side of (7) is strictly positive, the second and third terms are zero, while the fourth is strictly positive. Since β^* can yield no less profit than $\beta = 1$ by definition of it being an optimum, it must be that $\Pi_{IFI}(\beta^*, P^* \geq 0) > 0$. This contradicts the fact that $\Pi_{IFI}(\beta^*, P^*) = 0$ in equilibrium. Therefore, if it exists, $P^* \in (0, 1)$.
To show that P^* exists in the interval $(0, 1)$, we differentiate the right hand side of Π_{IFI} to show that profit is strictly increasing in P.

$$\begin{aligned}\frac{\partial\Pi_{IFI}}{\partial P} &= b\beta P\left[C'(\gamma-\beta P\gamma)\left(G-C(\gamma-\beta P\gamma)-\beta P\gamma\right)+\left(\overline{R}_f-C(\gamma-\beta P\gamma)\right)\beta\gamma\left(C'(\gamma-\beta P\gamma)-1\right)\right] \\ &\quad +(1-b)\left[G\gamma\left(\beta+(1-\beta)R_I\right)\right]+\gamma\left(\beta+(1-\beta)R_I\right) \qquad (8) \\ &> 0 \qquad (9)\end{aligned}$$

Where the inequality follows from the assumption that $G \geq C(\gamma-\beta P\gamma)-\beta P\gamma$ and the assumption that $C(x) \geq x\ \forall x \geq 0$ (which implies $C'(x) \geq 1$). Therefore, since profit is negative when $P^* \leq 0$ and positive when $P^* \geq 1$, and since profit is a (monotonically) increasing function of P^*, profit must equate to zero within $P^* \in (0, 1)$.

Step 2: Uniqueness

Assume the following holds: $\Pi_{IFI}(\beta^*, P_1^*) = 0$. Since we have already shown that profit is a strictly increasing function of P^*, then if $P_2^* > P_1^*$ ($P_2^* < P_1^*$) this implies $\Pi_{IFI}(\beta^*, P_2^*) > 0$ ($\Pi_{IFI}(\beta^*, P_2^*) < 0$). Therefore, since given P_2^* and P_1^* and $\Pi_{IFI}(\beta^*, P_1^*) = 0$ implies that $P_1^* = P_2^*$ must hold, our price is unique.

■

Proof of Lemma 3.
From the envelop theorem, we can ignore the effect that changes in b have on β when we evaluate the payoff at β^*. Plugging $\beta = \beta^*$ into (1) and taking the partial derivative with respect to b yields:

$$\begin{aligned} \left.\frac{\partial \Pi_{IFI}}{\partial b}\right|_{\beta=\beta^*} &= -\frac{\left(\overline{R}_f - C(\gamma - \beta^* P\gamma)\right)\left(C(\gamma - \beta^* P\gamma) + \beta^* P\gamma\right) + C(\gamma - \beta^* P\gamma)G + P\gamma G(\beta^* + (1-\beta^*)R_I)}{\overline{R}_f - \underline{R}_f} \\ &< 0 \end{aligned} \tag{10}$$

The inequality follows because $C(\cdot) > 0$ by assumption. Since the envelop theorem is a local condition and does not hold for large changes in b, it serves as an upper bound on the decrease in profits. It follows that an increase in b must be met with an increase in P otherwise the IFI would make negative profit and would not participate in the market.

■

Proof of Lemma 4. Since counterparty risk is defined as $\int_{\underline{R}_f}^{C(\gamma-\beta P\gamma)} f(\theta)d\theta$, we are interested in what happens to $C(\gamma - \beta^* P^*\gamma)$ as b changes.
We first focus on the case in which $\beta^* \in (0,1)$. We now take following partial derivative where we define $\frac{\partial \beta^*}{\partial b} \equiv \left.\frac{\partial \beta}{\partial b}\right|_{\beta=\beta^*}$ and $\frac{\partial P^*}{\partial b} \equiv \left.\frac{\partial P}{\partial b}\right|_{P=P^*}$.

$$\frac{\partial\left(\gamma - \beta^* P^*\gamma\right)}{\partial b} = -\gamma\left(\frac{\partial \beta^*}{\partial b}P^* + \beta^*\frac{\partial P^*}{\partial b}\right) \tag{11}$$

From Lemma 1 we know $\frac{\partial \beta^*}{\partial b} \geq 0$. As well, from Lemma 3 we know $\frac{\partial P^*}{\partial b} > 0$. Since $\beta^* \in (0,1)$ and $P^* > 0$ (from Lemma 2), it follows that:

$$\frac{\partial\left(\gamma - \beta^* P^*\gamma\right)}{\partial b} < 0 \tag{12}$$

Therefore, as b increases, counterparty risk decreases when $\beta \in (0,1)$. Next, consider the case of $\beta^* = 1$. Again, from Lemma 3 we know $\frac{\partial P^*}{\partial b} > 0$. Therefore, $\frac{\partial(\gamma-\beta^* P^*\gamma)}{\partial b} < 0$ regardless of whether $\frac{\partial \beta^*}{\partial b} = 0$ or $\frac{\partial \beta^*}{\partial b} > 0$. Thus, counterparty risk decreases when b decreases if $\beta^* = 1$.
It is obvious that if $\beta^* = 0$ there will be no change in counterparty risk by noting that $\beta^* P\gamma$ will be independent of b.

■

Proof of Proposition 2.

We begin by showing that there is no price $\tilde{P} < P_b^*$ such that the IFI can earn zero profit. It is straight-forward to see that $\Pi_{IFI}(\beta_b^*, P_b^*) = 0$ (where Π_{IFI} is defined by (1)) implies that $\Pi_{IFI}(\tilde{\beta}, \tilde{P}) \neq 0 \; \forall \; \tilde{\beta} \in [0,1]$ and for $\tilde{P} < P_b^*$.

Since Lemmas 1 and 2 show that with $(\beta_b^*,\ P_b^*)$, zero profit is attained, it must be with $\tilde{\beta} \in [0,1] \neq \beta_b^*$ and P_b^*, the IFI earns negative profits. It follows that with $\tilde{\beta}$ and $\tilde{P} < P_b^*$, the IFI also earns negative profits. Since the IFI must earn zero profits, $\tilde{P} \geq P_b^*$. This implies that $P_b^* \leq P_b^{sp}$.

The Proof now proceeds in 3 steps. Step 1 derives the first order condition for the planning problem. Step 2 assumes the equilibrium solution and derives an expression for $\frac{\partial P}{\partial \beta}$ from the IFI's zero profit condition. Step 3 shows that β^{pl} and P^{pl} must be greater than in the equilibrium case when $\beta^* < 1$. Since we need not specify a belief for this proof, it follows that the result holds regardless of there is separation of pooling of banks.

Step 1

The result is valid for both the ex-ante case and the case in which the types are known. We set up the problem in the case when the types are known; however, after the first order condition (13), we state a simple redefinition of the default probability parameter that will give us the ex-ante case. The profit for the bank (bk) of type $j \in \{S, R\}$) can be written as follows.

$$\Pi_{bk} \;=\; p_j R_B \gamma + \gamma(1-p_j)\int_{C(\gamma-\beta P\gamma)}^{\overline{R}_f} dF(\theta) - \gamma(1-p_j)Z\int_{\underline{R}_f}^{C(\gamma-\beta P\gamma)} dF(\theta) - \gamma P$$

In the planners case, P^{sp} is now endogenous and determined by $\Pi_{IFI}(\beta^{sp}, P^{sp}) = 0$ (where Π_{IFI} is defined by (1)). Using the uniform assumption on F yields the following first order condition.

$$\frac{\partial P}{\partial \beta} = \gamma C'(\gamma - \beta P\gamma)\left(P + \frac{\partial P}{\partial \beta}\beta\right)(1-p_j)(1+Z) \tag{13}$$

The left hand side represents the marginal cost of increasing β, while the right hand side represents the marginal benefit of doing so. Note that if we had derived this expression using the expected profit, then $(1-p_j) = \frac{1}{2}(2 - p_R - p_S)$.

Step 2

We show that if $\beta^{pl} = \beta^*$, then (13) cannot hold. We know that from the IFI's problem, the following must hold (see Lemma 1 for its derivation):

$$\begin{aligned} 0 \;=\;& \frac{b}{\overline{R}_f - \underline{R}_f}\left[C'(\gamma-\beta^*P^*\gamma)\left(G - C(\gamma-\beta^*P^*\gamma) - \beta^*P^*\gamma\right) + \left(\overline{R}_f - C(\gamma-\beta^*P^*\gamma)\right)\left(C'(\gamma-\beta^*P^*\gamma)-1\right)\right] \\ & + (1-b)\frac{G}{\overline{R}_f - \underline{R}_f}\left[-R_I\gamma P^* + \gamma P^*\right] + P^*\gamma(1-R_I) \end{aligned} \tag{14}$$

We now find an expression for $\left.\frac{\partial P}{\partial \beta}\right|_{\beta=\beta^*, P=P^*}$ by implicitly differentiating the equation $\Pi_{IFI}(\beta^*, P^*) =$

0.

$$
\begin{aligned}
0 &= (1-b)\left[\int_{-P\gamma(\beta+(1-\beta)R_I)}^{0} Gf(\theta)d\theta\right] - b\left[\int_{C(\gamma-\beta P\gamma)}^{\overline{R}_f} (C(\gamma-\beta P\gamma)+\beta P\gamma)\right] \\
&\quad -b\left[\int_{0}^{C(\gamma-\beta P\gamma)} Gf(\theta)d\theta\right] + P\gamma(\beta+(1-\beta)R_I) \qquad (15)
\end{aligned}
$$

Implicitly differentiating this equation to find $\frac{\partial P}{\partial \beta}$ yields the following.

$$
\begin{aligned}
A\left.\frac{\partial P}{\partial \beta}\right|_{\beta=\beta^*,P=P^*} &= (1-b)\frac{G}{\overline{R}_f-\underline{R}_f}\left[-R_I\gamma P^*+\gamma P^*\right] + P^*(\gamma(1-R_I) \\
&\quad +\frac{bP^*\gamma}{\overline{R}_f-\underline{R}_f}[C'(\gamma-\beta^*P^*\gamma)\left(G-C(\gamma-\beta^*P^*\gamma)-\beta^*P^*\gamma\right) \\
&\quad +\left(\overline{R}_f-C(\gamma-\beta^*P^*\gamma)\right)\left(C'(\gamma-\beta^*P^*\gamma)-1\right)] \qquad (16)
\end{aligned}
$$

Where we define:

$$
\begin{aligned}
A &= b\beta^*\gamma[C'(\gamma-\beta^*P^*\gamma)(C(\gamma-\beta^*P^*\gamma)+\beta^*P^*\gamma) - \left(\overline{R}_f-C(\gamma-\beta^*P^*\gamma)\right)\left(C'(\gamma-\beta^*P^*\gamma)-1\right) \\
&\quad +C'(\gamma-\beta^*P^*\gamma)G]. \qquad (17)
\end{aligned}
$$

It follows that $\left.\frac{\partial P}{\partial \beta}\right|_{\beta=\beta^*,P=P^*}=0$ since the right hand side of (16) is the FOC derived in Lemma 1 and must equate to 0 at the optimum, β^*.

Step 3

Substituting $\left.\frac{\partial P}{\partial \beta}\right|_{\beta=\beta^*,P=P^*}=0$ into (13) yields:

$$
0=\gamma C'(\gamma-\beta^*P^*\gamma)\left(P^*\right)(1-p_j)(1+Z), \qquad (18)
$$

which cannot hold since $\gamma>0$, $(1-p_j)>0$ and $Z>0$. Therefore, $\beta^{sp}\neq\beta^*$ and $P^{sp}\neq P^*$. To satisfy (13), it must be the case that $\beta^{sp}>\beta^*$. Since β^* was profit maximizing for the IFI, and with (β^*,P^*) the IFI earned zero profit, it follows that profit must be negative with (β^{sp},P^*). Therefore, $P^{sp}>P^*$ must hold so that the IFI earns zero profit when its investment choice is β^{sp}. This implies that the following must hold.

$$
\int_0^{C(\gamma-\beta^{sp}P^{sp}\gamma)} f(\theta)d\theta < \int_0^{C(\gamma-\beta^*P^*\gamma)} f(\theta)d\theta \qquad (19)
$$

Therefore, there is strictly less counterparty risk in the planners case than in the equilibrium case. It is obvious that if $\beta^*=1$ (the IFI invests everything in the liquid asset), the planner sets $\beta^{sp}=1$ and the counterparty risk does not change.

■

Proof of Lemma 5. Optimizing Π_{IFI}^{MB} choosing β yields the following first order condition (recall

F is assumed to be uniformly distributed):

$$
\begin{aligned}
0 \;=\; & \frac{1}{\overline{R}_f - \underline{R}_f} \int_0^{\beta^* PM} (-PM\gamma(1-R_I))\, Gdb(y) + \left(-PM\gamma(\beta^* + (1-\beta^*)R_I) + \beta^* PM\gamma - \underline{R}_f\right) GPM \\
& + \frac{1}{\overline{R}_f - \underline{R}_f} \int_{\beta^* PM}^{M} [-C'(y\gamma - \beta^* PM\gamma)(-PM\gamma)\left(-C(y\gamma - \beta^* PM\gamma) - \beta^* PM\gamma\right) \\
& + \left(\overline{R}_f - C(y\gamma - \beta^* PM\gamma)\right)\left(-C'(y\gamma - \beta^* PM\gamma)(-PM\gamma) - PM\gamma\right) \\
& + C'(y\gamma - \beta^* PM\gamma)(-PM\gamma)(-G)]db(y) \\
& - \frac{1}{\overline{R}_f - \underline{R}_f}\left[\left(\overline{R}_f - C(0)\right)\left(-C(0) - \beta^* PM\gamma\right) + \left(C(0) - \underline{R}_f\right)(-G)\right] PM + (1-R_I)PM\gamma \qquad (20)
\end{aligned}
$$

Recalling $C(0) = 0$ we simplify the above.

$$
\begin{aligned}
0 \;=\; & -\int_0^{\beta^* PM} \gamma(R_I - 1)Gdb(y) - PM\gamma(1-\beta^*)R_I G \\
& + \gamma \int_{\beta^* PM}^{M} [C'(y\gamma - \beta^* PM\gamma)\left(G - C(y\gamma - \beta^* PM\gamma) - \beta^* PM\gamma\right) \\
& + \left(\overline{R}_f - C(y\gamma - \beta^* PM\gamma)\right)\left(C'(y\gamma - \beta^* PM\gamma) - 1\right)]db(y) \\
& + \overline{R}_f \beta^* \gamma - \underline{R}_f G - \gamma(R_I - 1) \qquad (21)
\end{aligned}
$$

The SOC implies that the right hand side of (21) is decreasing in β^* so that our problem achieves a maximum. Define two belief distributions $b_1(y)$ and $b_2(y)$ such that $b_1(y) \geq b_2(y)\ \forall x$. As well, let $(\beta_1^*, b_1(y))$ solve the first order condition (20). Intuitively, moving from $b_1(y)$ to $b_2(y)$, mass shifts from the interval $[0, \beta^* PM]$ to $[\beta^* PM, M]$. Formally:

$$
\int_0^{\beta^* PM} db_1(y) \;>\; \int_0^{\beta^* PM} db_2(y) \qquad (22)
$$

$$
\int_{\beta^* PM}^{M} db_1(y) \;<\; \int_{\beta^* PM}^{M} db_2(y). \qquad (23)
$$

Since it is assumed that the FOC holds with $(\beta_1^*, b_1(y))$, given (22) and (23) that with $(\beta_1^*, b_2(y))$, it follows that β_1^* must increase for (21) to hold. In other words, the riskier the distribution of loans that the IFI insures, the more that it invests in the liquid asset.

To proceed we use a similar result to that of Lemma 3. It is straight forward to see that when the beliefs of default are higher (as in the risky case), so must the price of the contracts be higher (this can be shown in the same way that Lemma 3 was proved by showing that a net profit function is decreasing in the amount of risk in the loans). Next we find what happens to counterparty risk. What is different about the case of multiple banks is that the counterparty risk is defined relative to the number of banks that default: $\int_{\beta PM}^{M} \int_{\underline{R}_f}^{C(y\gamma - \beta PM\gamma)} dF(\theta)db(y)$.

In the case where the IFI puts more weight on the loans being risky ($p_A = r$), β^* and P^* increase, so that $C(\gamma - \beta P\gamma)$ decreases. Furthermore, since from the point of view of a bank the probability of a claim does not change, counterparty risk decreases as compared to when the IFI puts more weight on the loans being safe ($p_A = s$).

■